Complex Numbers & Quadratic Equations

1. For a positive integer n, find the value of $(1-i)^n\left(1-\frac{1}{i}\right)^n$

2. Evaluate $\sum_{n=1}^{13}(i^n+i^{n+1})$, where $n\in\mathbf{N}$.

3. If $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3 = x+iy$, then find (x, y).

4. If $\frac{(1+i)^2}{2-i}=x+iy$, then find the value of $x+y$.

5. If $\left(\frac{1-i}{1+i}\right)^{100} = a+ib$, then find (a, b).

6. If $a=\cos\theta+i\sin\theta$, find the value of $\frac{1+a}{1-a}$.

7. If $(1+i)\,z=(1-i)\,\bar{z}$, then show that $z=-i\bar{z}$.

8. If $z=x+iy$, then show that $z\bar{z}+2(z+\bar{z})+b=0$, where $b\in\mathbf{R}$, represents a circle.

9. If the real part of $\frac{\bar{z}+2}{\bar{z}-1}$ is 4, then show that the locus of the point representing z in the complex plane is a circle.

10. Show that the complex number z, satisfying the condition $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ lies on a circle.

11. Solve the equation $|z|=z+1+2i$.

12. If $|z+1| = z + 2(1+i)$, then find z.

13. If $\arg(z-1) = \arg(z+3i)$, then find $x-1 : y$. where $z = x + iy$

14. Show that $\left|\frac{z-2}{z-3}\right| = 2$ represents a circle. Find its centre and radius.

15. If $\frac{z-1}{z+1}$ is a purely imaginary number $(z \neq -1)$, then find the value of $|z|$.

16. z_1 and z_2 are two complex numbers such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$, then show that $z_1 = -\overline{z}_2$.

17. If $|z_1| = 1\ (z_1 \neq -1)$ and $z_2 = \frac{z_1 - 1}{z_1 + 1}$, then show that the real part of z_2 is zero.

18. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, then find $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$.

19. If $|z_1| = |z_2| = \ldots = |z_n| = 1$, then

show that $|z_1 + z_2 + z_3 + \ldots + z_n| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \ldots + \frac{1}{z_n}\right|$.

20. If for complex numbers z_1 and z_2, $\arg(z_1) - \arg(z_2) = 0$, then show that $|z_1 - z_2| = |z_1| - |z_2|$

21. Solve the system of equations $\text{Re}(z^2) = 0$, $|z| = 2$.

22. Find the complex number satisfying the equation $z + \sqrt{2}\ |(z+1)| + i = 0$.

23. Write the complex number $z = \frac{1-i}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ in polar form.

24. If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then show that $\overline{z}w = -i$.

25. What is the conjugate of $\frac{2-i}{(1-2i)^2}$?

26. If $|z_1| = |z_2|$, is it necessary that $z_1 = z_2$?

27. If $\frac{(a^2+1)^2}{2a-i} = x + iy$, what is the value of $x^2 + y^2$?

28. Find z if $|z| = 4$ and $\arg(z) = \frac{5\pi}{6}$.

29. Find $\left|(1+i)\frac{(2+i)}{(3+i)}\right|$

30. Find principal argument of $(1 + i\sqrt{3})^2$.

31. Where does z lie, if $\left|\frac{z-5i}{z+5i}\right| = 1$.

32. $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other for:

(A) $x = n\pi$ (B) $x = \left(n+\frac{1}{2}\right)\frac{\pi}{2}$

(C) $x = 0$ (D) No value of x

33. The real value of α for which the expression $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely real is :

(A) $(n+1)\frac{\pi}{2}$ (B) $(2n+1)\frac{\pi}{2}$

(C) $n\pi$ (D) None of these, where $n \in \mathbf{N}$

34. If $z = x + iy$ lies in the third quadrant, then $\frac{\bar{z}}{z}$ also lies in the third quadrant if

(A) $x > y > 0$ (B) $x < y < 0$

(C) $y < x < 0$ (D) $y > x > 0$

35. The value of $(z + 3)(\bar{z} + 3)$ is equivalent to

(A) $|z+3|^2$ (B) $|z-3|$

(C) $z^2 + 3$ (D) None of these

36. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then

(A) $x = 2n+1$ (B) $x = 4n$

(C) $x = 2n$ (D) $x = 4n + 1$, where $n \in \mathrm{N}$

37. A real value of x satisfies the equation $\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta$ $(\alpha, \beta \in \mathbf{R})$ if $\alpha^2 + \beta^2 =$

(A) 1 (B) -1 (C) 2 (D) -2

38. Which of the following is correct for any two complex numbers z_1 and z_2?

(A) $|z_1 z_2| = |z_1||z_2|$ (B) $\arg(z_1 z_2) = \arg(z_1).\arg(z_2)$

(C) $|z_1 + z_2| = |z_1| + |z_2|$ (D) $|z_1 + z_2| \geq |z_1| - |z_2|$

39. The point represented by the complex number $2 - i$ is rotated about origin through

an angle $\frac{\pi}{2}$ in the clockwise direction, the new position of point is:

(A) $1 + 2i$ (B) $-1 - 2i$ (C) $2 + i$ (D) $-1 + 2i$

40. Let $x, y \in \mathbb{R}$, then $x + iy$ is a non real complex number if:

(A) $x = 0$ (B) $y = 0$ (C) $x \neq 0$ (D) $y \neq 0$

41. If $a + ib = c + id$, then

(A) $a^2 + c^2 = 0$ (B) $b^2 + c^2 = 0$

(C) $b^2 + d^2 = 0$ (D) $a^2 + b^2 = c^2 + d^2$

42. The complex number z which satisfies the condition $\left|\frac{i+z}{i-z}\right| = 1$ lies on

(A) circle $x^2 + y^2 = 1$ (B) the x-axis

(C) the y-axis (D) the line $x + y = 1$.

43. If z is a complex number, then

(A) $|z^2| > |z|^2$ (B) $|z^2| = |z|^2$

(C) $|z^2| < |z|^2$ (D) $|z^2| \geq |z|^2$

44. $|z_1 + z_2| = |z_1| + |z_2|$ is possible if

(A) $z_2 = \overline{z_1}$ (B) $z_2 = \frac{1}{z_1}$

(C) $\arg(z_1) = \arg(z_2)$ (D) $|z_1| = |z_2|$

45. The real value of θ for which the expression $\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is a real number is:

(A) $n\pi + \frac{\pi}{4}$ (B) $n\pi + (-1)^n \frac{\pi}{4}$

(C) $2n\pi \pm \frac{\pi}{2}$ (D) none of these.

46. The value of $\arg(x)$ when $x < 0$ is:

(A) 0 (B) $\frac{\pi}{2}$

(C) π

(D) none of these

47. If $f(z) = \frac{7-z}{1-z^2}$, where $z = 1 + 2i$, then $|f(z)|$ is

(A) $\frac{|z|}{2}$

(B) $|z|$

(C) $2|z|$

(D) none of these.